Enrollment No: _____   Exam Seat No: _____

C.U.SHAH UNIVERSITY
Summer Examination-2016

Subject Name:Mathematics-II

Subject Code:4SC02MTC1   **Branch: B.Sc.(All)**

Semester: 2   Date: 13/05/2016   Time: 10:30 To 1:30   Marks: 70

Instructions:
    (1) Use of Programmable calculator & any other electronic instrument is prohibited.
    (2) Instructions written on main answer book are strictly to be obeyed.
    (3) Draw neat diagrams and figures (if necessary) at right places.
    (4) Assume suitable data if needed.

Q-1	**Attempt the following questions:**	**(14)**
a)	State De'Moivre's theorem.	**(02)**
b)	Prove that $\cosh^2 x - \sinh^2 x = 1$.	**(02)**
c)	Evaluate: $\int_0^{\frac{\pi}{2}} \cos^6 x \; dx$.	**(02)**
d)	Solve: $(D^2 + 6D + 9)y = 0$.	**(02)**
e)	$\frac{1}{f(D)}(e^{mx} x^2) = e^{mx} \frac{1}{f(D+m)} x^2$. Determine whether the statement is true or false?	**(01)**
f)	Define: Oscillatory Sequence.	**(01)**
g)	Find polar form of $1 - i\sqrt{3}$.	**(01)**
h)	Write a condition that the general equation of second degree represents a cone.	**(01)**
i)	Any line on the surface of a cylinder is called its_____.	**(01)**
j)	$\frac{1}{D-m} X = e^{-mx} \int X e^{mx} \; dx$. Determine whether the statement is true or false?	**(01)**

Attempt any four questions from Q-2 to Q-8

Q-2	**Attempt all questions**	**(14)**
a)	Solve:$x^6 - 1 = 0$.	**(05)**
b)	Expand $\frac{sin6\theta}{sin\theta}$ in terms of increasing power of $\cos\theta$.	**(05)**
c)	Prove that $\tan(i \log \frac{a-ib}{a+ib}) = \frac{2ab}{a^2-b^2}$.	**(04)**
Q-3	**Attempt all questions**	**(14)**
a)	Find equation of cylinder whose generators intersect the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0; z = 0$.	**(05)**
b)	Prove that $\cos^8 \theta = \frac{1}{128}[\cos 8\theta + 8cos6\theta + 28 \cos 4\theta + 56 \cos 2\theta + 35]$.	**(05)**
c)	Prove that $\sinh^{-1}(\tan x) = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$.	**(04)**

Q-4   **Attempt all questions**   **(14)**

a) Find equation of cone whose vertex is the point $(1, 1, 1)$ and whose guiding curve   **(05)**
is $x^2 + z^2 = 4, y = 0$.

b) Prove that $\lim_{n \to \infty} \sqrt[n]{n} = 1$.   **(05)**

c) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 = 25$,   **(04)**
whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

Q-5   **Attempt all questions**   **(14)**

a) Check whether the equation   **(05)**
$x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6xz + 8x - 19y - 2z - 20 = 0$ represent a
cone with vertex $(1, -2, 3)$.

b) Prove that equation of right circular cylinder whose axis is the line   **(05)**
$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r is $(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 -$
$\frac{[l(x-\alpha)+m(y-\beta)+n(z-\gamma)]^2}{l^2+m^2+n^2} = r^2$

c) Prove that $\lim_{n \to \infty} \frac{1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}}{n} = 0$.   **(04)**

Q-6   **Attempt all questions**   **(14)**

a) Prove that $\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$.   **(05)**

b) Solve: $y'' - 5y' + 6y = 0; y(1) = e^2, y'(1) = 3e^2$.   **(05)**

c) Evaluate: $i)$ $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^8 x \, dx$, $ii)$ $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x \, dx$.   **(04)**

Q-7   **Attempt all questions**   **(14)**

a) Solve: $3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - y = e^{\frac{x}{2}} + 2e^{3x}$.   **(05)**

b) Solve: $(D^3 + 3D^2 + 2D)y = x^2$.   **(05)**

c) Solve: $(D^3 + 1)y = 0$.   **(04)**

Q-8   **Attempt all questions**   **(14)**

a) Solve: $(x^2D^2 - 3xD + 4)y = x^2$.   **(05)**

b) Solve: $\frac{dx}{dt} = 5x + y, \frac{dy}{dt} = y - 4x$.   **(05)**

c) Identify the given surface $9x^2 + 4y^2 - 9z^2 - 18x - 8y - 18z = 32$.   **(04)**